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Theories and models for $1/f^{\beta}$ noise in human movement science

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ABSTRACT

Human motor behavior is often characterized by long-range, slowly decaying serial correlations or $1/f^{\beta}$ noise. Despite its prevalence, the role of the $1/f^{\beta}$ phenomenon in human movement research has been rather modest and unclear. The goal of this paper is to outline a research agenda in which the study of $1/f^{\beta}$ noise can contribute to scientific progress. In the first section of this article we discuss two popular perspectives on $1/f^{\beta}$ noise: the nomothetic perspective that seeks general explanations, and the mechanistic perspective that seeks domain-specific models. We believe that if $1/f^{\beta}$ noise is to have an impact on the field of movement science, researchers should develop and test domain-specific mechanistic models of human motor behavior. In the second section we illustrate our claim by showing how a mechanistic model of $1/f^{\beta}$ noise can be successfully integrated with currently established models for rhythmic self-paced, synchronized, and bimanual tapping. This model synthesis results in a unified account of the observed long-range serial correlations across a range of different tasks.

1. Introduction

In the field of human movement science, researchers often quantify performance in terms of its accuracy and its consistency. For instance, consider a task in which a participant first listens to the beat of a metronome. When the metronome stops, the participant sets out to reproduce the metronome's rhythm by tapping a finger. In this so-called continuation tapping task, performance may be

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measured by the average inter-beat interval of the participant relative to that of the metronome. Performance may also be measured by the variability of the participant's inter-beat intervals; high variability is then associated with low consistency and poor performance.

Despite the fact that accuracy and consistency are important global indicators of successful performance, neither of the two measures directly relates to the trial-to-trial dynamics of the system under investigation. In fact, the standard statistical analysis of consistency tacitly assumes that trial-to-trial dynamics are absent and that consecutive behaviors are unrelated. This tacit assumption is clearly false, however, as previous research has convincingly demonstrated the presence of a strong correlation between consecutive behaviors. Moreover, the nature of this correlation has been used to constrain theories of human movement production. In particular, several models for finger tapping were developed to accommodate the negative lag 1 correlation observed in consecutive taps (e.g., the Wing and Kristofferson tapping model and its extensions; Vorberg & Schulze, 2002; Vorberg & Wing, 1996; Wing & Kristofferson, 1973).

Thus, it has long been known that human movement production leads to robust serial correlations that can inform us about the trial-to-trial dynamics of the underlying system. It is only recently, however, that researchers have started to examine more closely the specific kind of robust serial correlations observed in human movement production. Much of this recent work suggests that the serial correlations may be part of a special class known as $1/f^{\beta}$ noise. This particular class of serial correlations occurs throughout many widely different systems and reflects the presence of fractal features such as *self-similarity* and *scale-invariance*. In addition, the presence of $1/f^{\beta}$ noise implies that the serial correlations decay so slowly that the generating system is called persistent or *long-range dependent*. In the field of human movement science, $1/f^{\beta}$ noise has been found in human force production (Gilden, 2001; Wing, Daffertshofer, & Pressing, 2004), unimanual rhythmic movement (e.g., Chen, Ding, & Kelso, 1997; Chen, Ding, & Kelso, 2001; Delignières, Lemoine, & Torre, 2004b; Ding, Chen, & Kelso, 2002; Gilden, Thornton, & Mallon, 1995; Lemoine, Torre, & Delignières, 2006; Yoshinaga, Miyazima, & Mitake, 2000; Yulmetyev, Emelyanova, Hänggi, Gafarov, & Prokhorov, 2002; but see Pressing and Jolley-Rogers (1997), and the dynamics of bimanual coordination (Torre, Delignières, & Lemoine, 2007). In his keynote lecture at the 2007 European Workshop on Movement Science, Jeffrey Hausdorff argued for the diagnostic value of $1/f^{\beta}$ noise in the evaluation of human walking patterns (Hausdorff, 2007; see also Ashkenazy, Hausdorff, Ivanov, & Stanley, 2002; Hausdorff, Peng, Ladin, Wei, & Goldberger, 1996; West & Scafetta, 2003).

In human movement science, the phenomenon of $1/f^{\beta}$ noise can be approached from at least two different perspectives. The nomothetic perspective seeks general principles that explain the existence of $1/f^{\beta}$ noise across a range of different systems and behaviors. Proponents of the nomothetic perspective often explain the presence of $1/f^{\beta}$ noise by referring to the dynamic, self-organizing characteristics of the human nervous system (e.g., van Orden, Holden, & Turvey, 2003). The mechanistic perspective seeks domain-specific explanations for the existence of $1/f^{\beta}$ noise. Proponents of the mechanistic perspective explain the presence of $1/f^{\beta}$ noise by concrete modeling of the underlying processes that supposedly give rise to the serial correlations in the system under study (e.g., Ashkenazy et al., 2002; Delignières, Torre, & Lemoine, 2008; Wagenmakers, Farrell, & Ratcliff, 2004).

The primary goal of this article is to discuss the strengths and limitations of the nomothetic and mechanistic perspectives on $1/f^{\beta}$ noise. Note that we believe that both approaches have merit, and a direct comparison between the two is hampered by the fact that the strengths of the nomothetic perspective corresponds to the limitations of the mechanistic perspective, and vice versa. Thus, the two approaches are best seen as complementary. Nevertheless, we hope to demonstrate that the choice of perspective influences the research agenda in important ways.

While it is true that nomothetic accounts have famously contributed to the understanding of what $1/f^{\beta}$ noise tells about a system's underlying dynamics, our focus here is on the possible contribution of the mechanistic perspective. We hope to convince the reader that the mechanistic perspective on $1/f^{\beta}$ noise can be useful for theories of human movement production.

In the first part of this article we outline the nomothetic and the mechanistic perspectives on the phenomenon of $1/f^{\beta}$ noise. We argue that domain-specific models of the data-generating process are useful not only to explain the observed behavior but also to bridge the gap between the observed data and the existing theories of $1/f^{\beta}$ noise (cf. Gisiger, 2001; Jensen, 1998; Wagenmakers, Farrell, & Ratcliff, 2005).

In the second part of this article we exemplify our line of reasoning by outlining a mechanistic model for $1/f^{\beta}$ noise in self-paced tapping, and by showing how this model can be extended to synchronization tapping and bimanual tapping. This work illustrates how $1/f^{\beta}$ noise can drive theoretical progress and result in a unified framework for the modeling of tapping tasks that are superficially quite different.

2. Signature of a $1/f^{\beta}$ noise process

In order to make this article self-contained, we first briefly discuss the signature of a $1/f^{\beta}$ process and how it differs from more mundane processes. For concreteness, consider a task in which the participant has to estimate a one-second time interval and do this repeatedly for 400 uninterrupted trials without any feedback. Fig. 1, panel A shows an example data set (Wagenmakers, Grünwald, & Steyvers, 2006). From the raw data, it is immediately evident that this data set displays considerable positive trial-to-trial correlation; that is, if the participant's estimate is relatively high at trial n, it is likely to still be relatively high at trial $n + 1$ (i.e., lag-1 autocorrelation). Fig. 1, panel B plots all autocorrelations up to lag 25. This autocorrelation function shows that the correlation between different estimation attempts decreases with lag, but at a relatively slow rate. Moreover, a substantial autocorrelation remains even after more than 20 intervening estimation attempts. Finally, Fig. 1, panel C shows the log–log power spectrum. In order to obtain this plot, the raw data from panel A are first decomposed into their constituent sine and cosine waves. Next, the frequency of the waves is plotted against their squared amplitude. A visual impression of the power spectrum shows that the low-frequency waves have the highest amplitude, which suggests that an increase of the measurement scale leads to the detection of more and more low frequency waves with high amplitude.

The features of the time series in Fig. 1 are characteristic of a $1/f^{\beta}$ process (for details see Beran, 1994; Rangarajan & Ding, 2003). First, a $1/f^{\beta}$ process has an autocorrelation that decays so slowly that its sum does not converge to a finite number. Specifically, the correlation C with k intervening trials is given by a power function, $C(k) = |k|^{-\gamma}$, with γ between 0 and 1. This means that the process is *long-range dependent*. Second, the log–log power spectrum of a $1/f^{\beta}$ process is linear with slope $-\beta$, where β

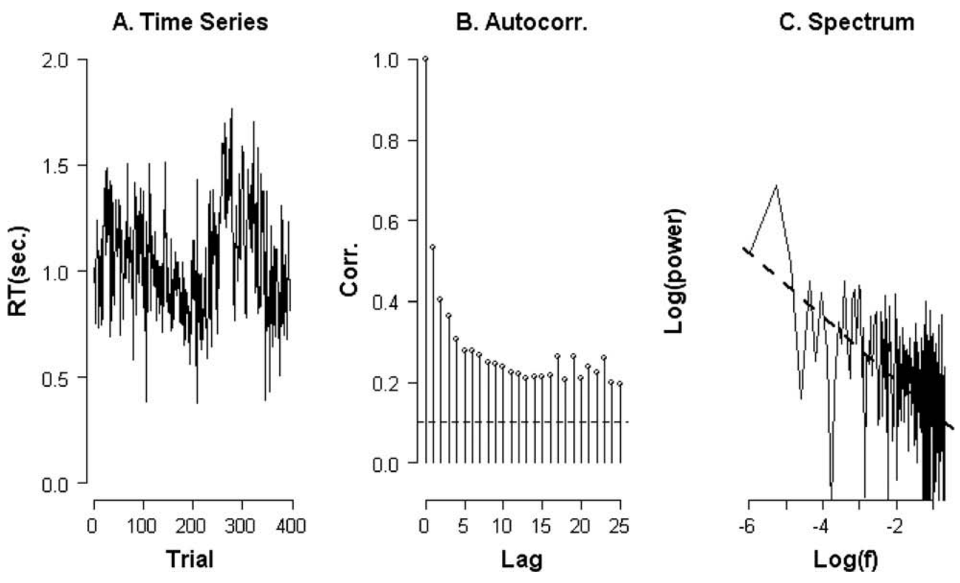

Fig. 1. Example of a $1/f$ process. Left panel: 400 consecutive attempts at estimating a one-second time interval without any feedback (Wagenmakers et al., 2006); center panel: the autocorrelation function for the time series from the left panel; right panel: the log–log power spectrum for the time series from the left panel. See text for details.

is usually taken to range from 0.5 to 1.5. Note that a sequence of independent normal deviates shows a slope of 0, whereas the cumulative sum of normal deviates (i.e., a random walk) shows a slope of -2. A third feature of a $1/f^{\beta}$ process is that it is *self-similar*: the statistical properties of the time series are the same regardless of the scale of measurement, and hence the process lacks a characteristic time scale (e.g., Maylor, Chater, & Brown, 2001).

The features that characterize the $1/f^{\beta}$ process are special. Traditional ARMA time series models, for instance, decompose the time series X_t as $X_t = \sum_{r=1}^{p} \phi_r X_{t-r} + \varepsilon_t + \sum_{r=1}^{q} \theta_r \varepsilon_{t-r}$, where ε is white noise and p and q indicate the order of the autoregressive and moving average components, respectively. In contrast to the $1/f^{\beta}$ process, an ARMA process has a characteristic time scale, a log–log power spectrum that levels off at the low frequencies, and an autocorrelation function that decays relatively quickly (i.e., ARMA models are short-range dependent). Thus, at least in theory, the $1/f^{\beta}$ process is quite different from the standard type of time series process (Thornton & Gilden, 2005; Wagenmakers et al., 2004).

The $1/f^{\beta}$ process is special, not just because of its unique features, but also because the origins of the process are presently not well understood. In particular, it is not obvious how to construct a general model that produces perfect $1/f^{\beta}$ noise – most models require either detailed knowledge of the specific application, account for only a very specific range of values for β (i.e., $\beta = 1$), or break down as the number of observations increases.

The air of mystique that surrounds $1/f^{\beta}$ noise – a special process whose precise origin is unknown – becomes even more intense when one considers that $1/f^{\beta}$ noise is found almost everywhere: examples of $1/f^{\beta}$ noise include electric current in transistors, water levels in the river Nile, the size of tree rings, brain activity as recorded by magnetoencephalogram, the stock market, music, and speech (e.g., Handel & Chung, 1993; Hosking, 1984; Hurst, 1951; Novikov, Novikov, Shannahoff-Khalsa, Schwartz, & Wright, 1997; Voss & Clarke, 1975; Wolf, 1978).[1]

In cognitive psychology, evidence for long-range dependence was recently found in a range of tasks such as mental rotation, lexical decision, speeded visual search, estimation of distance, estimation of rotation, estimation of force, estimation of time, simple reaction time, speech production, and word naming (Gilden, 1997, 2001; Gilden & Hancock, 2007; Gilden et al., 1995; Kello, Anderson, Holden, & van Orden, 2008; Kello, Beltz, Holden, & van Orden, 2007; van Orden et al., 2003; but see Farrell, Wagenmakers, & Ratcliff, 2006). Long-range dependence has also been reported in day-to-day fluctuations in self-esteem (Delignières, Fortes, & Ninot, 2004a), in the temporal dynamics of tics in Gilles de la Tourette syndrome (Peterson & Leckman, 1998), and in day-to-day fluctuations in selfmood of bipolar patients (Gottschalk, Bauer, & Whybrow, 1995).

Despite its special features, mysterious origin, and ubiquity throughout a range of different systems, the phenomenon of $1/f^{\beta}$ noise has often been ignored, both in experimental psychology and in human movement science, perhaps because of the belief that the phenomenon is inconsequential and erratic. However, Gilden (2001) has showed that, at least in certain experimental tasks, $1/f^{\beta}$ noise account for a large proportion of the observed variance, a proportion that is substantially larger than that caused by standard experimental manipulations. In addition, the intensity of $1/f^{\beta}$ noise (i.e., the slope β) is known to change systematically as a function of certain experimental manipulations (e.g., Chen et al., 2001; Hausdorff et al., 1996; Jordan, Challis, & Newell, 2006, 2007; Madison, 2001, 2004). Therefore, it appears as if – at least in domains where the phenomenon is well-established, such as in human movement science – the phenomenon of $1/f^{\beta}$ noise deserves more attention than it has previously received.

3. Two complementary perspectives on $1/f^{\beta}$ noise in psychological research

As mentioned in the introduction, the phenomenon of $1/f^{\beta}$ noise has been approached from either a nomothetic or a mechanistic perspective. In order to appreciate the differences in the interpretation of $1/f^{\beta}$ noise, it is important to clearly distinguish between these two perspectives without falling into caricatured oppositions. Both perspectives have value, and they are perhaps best seen as

[1] A comprehensive bibliography of $1/f$ noise is maintained by Wentian Li at http://www.nslij-genetics.org/wli/1fnoise/.

complementary rather than competing. In order to avoid any ambiguity from the outset, let us briefly clarify the distinction between the two perspectives. The nomothetic perspective focuses on the ubiquity of $1/f^{\beta}$ noise, and searches for general principles that account for its occurrence. The mechanistic perspective departs from the idea that, depending on the behavior under study, different causal mechanisms may be responsible for $1/f^{\beta}$ noise. In the mechanistic perspective, whatever causal mechanism one prefers, that mechanism needs to be modeled in enough detail to allow a quantitative test to data. Note that the distinction between the nomothetic and the mechanistic perspectives of $1/f^{\beta}$ noise is not the same as the distinction between verbal and mathematical accounts; indeed, nomothetic accounts are usually based on a substantive amount of mathematical formalization.

We acknowledge that the distinction between nomothetic and mechanistic perspectives on $1/f$ noise may not be one that is all-or-none – hybrid perspectives do exists, and there may be considerable shades of grey in between our black and white distinction. Nevertheless, we feel the paradigmatic distinction that we draw has face validity, as it maps on to different research agendas that we will outline in more detail below.

3.1. The nomothetic perspective on $1/f^{\beta}$ noise

Researchers with a nomothetic perspective on $1/f^{\beta}$ noise promote a general explanation of $1/f^{\beta}$ noise. These researchers often stress the fact that the $1/f^{\beta}$ phenomenon is ubiquitous (e.g., Gilden, 2001; Kello et al., 2008; van Orden et al., 2003), arguing that it is futile to try to explain $1/f^{\beta}$ noise using models that apply only in a limited domain, or, in other words, that "$1/f$ scaling is too pervasive to be idiosyncratic" (Kello et al., 2008). The nomothetic tradition focuses, first, on empirically demonstrating the presence of $1/f^{\beta}$ noise, and, second, on explaining the presence of $1/f^{\beta}$ noise by referring to the behavior of complex systems, multiple interacting sub-systems, emergent dynamics, metastability, structure at different time scales, and self-organized criticality. Occasionally, the proponents of the nomothetic account argue that the framework of cognitive psychology should be abandoned in favor of the framework of nonlinear dynamical systems theory (e.g., van Orden et al., 2003).

The main attraction and an obvious strength of the nomothetic perspective is that it proposes general explanations of $1/f^{\beta}$ noise. Such a general explanation could potentially solve the mystery of $1/f^{\beta}$ noise by revealing what it is that the very different systems that display $1/f^{\beta}$ noise have in common. Here we discuss two general explanations of $1/f^{\beta}$ noise; *self*-organized criticality and aggregation of short-range processes with different time scales.

3.1.1. Self-organized criticality

In order to explain the ubiquitous presence of $1/f^{\beta}$ noise, the physicist Per Bak and his colleagues developed the concept of self-organized criticality (SOC; e.g., Bak, 1996; Bak, Tang, & Wiesenfeld, 1987, Paczuski, Maslov, & Bak, 1996; but see Jensen (1998), and Jensen, Christensen, & Fogedby, 1989; for a recent review with respect to biological systems, see Gisiger (2001); see also Sornette, 2000). Systems with SOC can display $1/f^{\beta}$ noise, albeit only under specific conditions and only for specific dependent variables. The concept of SOC is aptly illustrated by the behavior of a particular pile of sand (e.g., Jensen, 1998; Wagenmakers et al., 2005). This pile of sand is constrained by two orthogonal walls, so that it is bunched up in a corner. At random positions along the walls, new grains of sand are continually dropped onto the pile. At some point, the local slope of the sand pile exceeds a certain threshold and grains of sand are transported downhill until the local slope is again below threshold. This mechanism can cause avalanches of different sizes; when several adjacent slopes are near their local threshold, a single added grain of sand can lead to a chain reaction that brings about a cascade of avalanches.

The above pile of sand is said to self-organize to reach a critical state. Once in this state, small perturbations (i.e., single grains of sand) may have large consequences (i.e., a cascade of avalanches). Similar models have been proposed for evolution (e.g., Bak & Sneppen, 1993; but see Davidsen and Lüthje (2001), forest fires (e.g., Malamud, Morein, & Turcotte, 1998), earthquakes (e.g., Davidsen & Paczuski, 2002; Davidsen & Schuster, 2000, Davidsen & Schuster, 2002), and populations of neurons (da Silva, Papa, & de Souza, 1998; Usher, Stemmler, & Olami, 1995). The above models all assume that the system of interest is gradually pushed toward a threshold, and that there are dominant interactions between many of the system's individual units. Hence, Jensen (1998) termed these kinds of models "slowly driven, interaction-dominated threshold systems" (p. 126).

Proponents of the nomothetic account often point to the advantages of SOC for neural networks: A neural network that is in a state of criticality is able to quickly reorganize and swiftly adapt to new situations (Alstrøm & Stassinopoulos, 1995; Bak & Chialvo, 2001; Chialvo & Bak, 1999; Linkenkaer-Hansen, Nikouline, Palva, & Ilmoniemi, 2001). Thus, it is argued, the presence of $1/f^{\beta}$ noise in human cognition or human motor behavior may signal SOC as the underlying design principle. This design principle is beneficial because it allows the system to adjust to changes in environmental demands.

The theory of SOC is elegant and attractive. Its main weakness, as was hinted at above, is that SOC systems generate $1/f^{\beta}$ noise only under specific conditions and only for specific dependent variables. For instance, the total mass of the self-organizing pile of sand shows $1/f^{\beta}$ noise across a wide range of frequencies (Jensen, 1998, pp. 30–42), but this only happens when the new grains are added along the two orthogonal walls. Surprisingly, the pile of sand does not show $1/f^{\beta}$ noise when the grains of sand are added to random positions on the interior of the pile (see Jensen, 1998, p. 42). Also, certain piles do not generate $1/f^{\beta}$ noise when they are made up of sand, but do generate $1/f^{\beta}$ noise when they are made up of rice (for details, see Jensen, 1998). The dramatic impact of such design details highlights the need for specific models of the underlying process. For the proponents of the nomothetic account, the lack of robustness with which SOC systems generate $1/f^{\beta}$ noise may take away some of considerable appeal.

3.1.2. Aggregation of short-range processes with different time scales

It has been known for a long time that $1/f^{\beta}$ noise can be produced by summing component short-range processes with different characteristic time scales (see, e.g., Gardner, 1978; Jensen, 1998, p. 9; van der Ziel, 1950; Wagenmakers et al., 2004, pp. 603–605). For example, consider a time series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$, where X denotes the observed output of the system as a whole, and the Ys indicate the output of individual subcomponents. The Ys may or may not be observed. Suppose that all Y are switching series, that is, they retain their value on the previous trial with probability $p = \exp(-1/\tau)$. When Y_1 is a quickly changing process with $\tau = 1$, Y_2 is an intermediate process with $\tau = 10$, and Y_3 is a slowly changing process with $\tau = 100$, the composite series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$ has structure at three different time scales – for a time series of about 1000 observations, this yields almost perfect $1/f^{\beta}$ noise (Wagenmakers et al., 2004, Fig. 12).

Several models for $1/f^{\beta}$ noise in complex natural systems have exploited the above principle of aggregation. In biology, the model by Hausdorff and Peng (1996) assumes that heart rate fluctuations are subject both to relatively quick adjustments (e.g., beat-by-beat) via the autonomic nervous system, and to relatively slow adjustments (e.g., circadian rhythms) via hormonal systems. Ivanov, Nunes Amaral, Goldberger, and Stanley (1998) proposed a model for the regulation of heart rate through homeostasis that is based on similar principles. In cognitive psychology, Ward (2002) has promoted the principle of aggregation by distinguishing between fast fluctuating preconscious processes, slowly fluctuating conscious processes, and unconscious processes that operate on an intermediate time scale. In the movement sciences, Pressing (1999) has applied the principle of aggregation to explain the presence of $1/f^{\beta}$ noise in synchronous tapping.

The main benefits of explaining $1/f^{\beta}$ noise by aggregation of component processes is that such an explanation is conceptually transparent (i.e., it demystifies the $1/f^{\beta}$ phenomenon), and focuses attention on the latent processes that influence the system's behavior. One main drawback of explaining $1/f^{\beta}$ noise by aggregation is that, as the length of the time series increases, more and more short-range processes need to be invoked to keep the spectrum from flattening at the low frequencies. In the limit of many samples, the aggregation approach is thus not very parsimonious. In addition, it could be argued that, in order to generate $1/f^{\beta}$ noise, the time scales of the component processes need to coordinate in just the right way – for instance, the above time series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$ only shows $1/f^{\beta}$ noise if the time scales for the Ys are sufficiently different.

This latter concern was addressed by Granger ((1980); see also Robinson (1978), who showed that "blind" aggregation of short-range processes can also produce $1/f^{\beta}$ noise. For instance, assume that the observed behavior X_t of a given system is just the sum of infinitely many component series, $X_t = \sum_{i=1}^{\infty} Y_t^{(i)}$. Further assume that each individual Y component is a first-order autoregressive process,

$Y_t^{(k)} = \phi^{(k)} Y_{t-1}^{(k)} + \varepsilon_t$, and let $\phi^{(k)}$ be sampled from a beta distribution with sufficient mass near 1. Then the observed behavior X_t shows $1/f^\beta$ noise. This explanation of $1/f^\beta$ noise is popular in the fields of economics and finance (see, e.g., Baillie (1996), but it can easily be extended to human cognition and motor control; one only needs to make the plausible assumption that the observed behavior is jointly determined by many independent groups of neurons, each with their own different autoregressive decay parameter (cf. Chen et al., 2001; Ding et al., 2002).

3.1.3. Limitations of the nomothetic perspective

The nomothetic perspective is valuable in that it tries to formulate general explanations for a ubiquitous phenomenon. That is, the ubiquitous finding of $1/f^\beta$ noise in human coordination may be accounted for by the general hypothesis that the human nervous system displays self-organized criticality, just as sand piles and forest fires do. The counterpart of this level of generality is that it is to some extent accompanied by a detachment from the singularity of the phenomenon of interest. For instance, the skeptical researcher may wonder what exactly we can learn from the presence of $1/f^\beta$ noise in, say, human motor coordination, other than that human coordination shares certain statistical similarities with sand piles and forest fires. The mere fact that $1/f^\beta$ noise occurs throughout nature does not make the phenomenon psychologically meaningful (e.g., Uttal, 2003).

3.2. The mechanistic perspective on $1/f^\beta$ noise

Proponents of the mechanistic perspective explain the presence of $1/f^\beta$ noise by concrete modeling of underlying processes. These researchers point out that their purpose is to account for the workings of a particular psycho-physiological system, not solely to account for the $1/f^\beta$ noise the system may display. From this perspective, $1/f^\beta$ noise is just another finding that provides a useful constraint for modeling.

The need for concrete models, the proponents point out, is further motivated by some of the following concerns. First, concrete models produce concrete questions and concrete answers, as they are closely related to the phenomenon of interest. Using concrete models, the importance of discovering $1/f^\beta$ noise for, say, human motor coordination becomes much clearer. Second, concrete models are experimentally testable and falsifiable. Third, the majority of empirical studies do not find that experimental manipulations cause a discrete transition from pure $1/f^\beta$ noise (i.e., $\beta = 1$) to uncorrelated white noise (i.e., $\beta = 0$); instead, experimental manipulations often lead to a gradual shift of the exponent, such that the intensity of the long-range dependence might change from, say, $\beta = 0.8$–0.4. The explanation of this pattern of results requires a domain-specific model that takes into account the singularity of the observed behavior. Finally, there is no a priori reason why long-range and short-range dependence should be mutually exclusive, and the observed serial correlation are likely the result of both. In these cases, statistical models are needed to separate the long-range from the short-range components.

Here we discuss two accounts of $1/f^\beta$ noise that have been implemented as concrete models for specific tasks – the hopping model and the shifting strategy model.

3.2.1. The hopping model

The hopping model was developed to account for the long-range correlations observed in stride intervals in human gait (Ashkenazy et al., 2002; West & Scafetta, 2003). The model builds on previous theories of human gait dynamics. A central pattern generator, regrouping firing neuron centers, has been assumed to be responsible for the gait pattern, and the stride frequency in particular (Collins & Richmond, 1994; Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995). The dynamics of gait cycles has been modeled by a forced van der Pol oscillator (Guckenheimer & Holmes, 2002), so that the firing intensity of the neural centers was assumed to determine the eigenfrequency of the oscillator through its linear stiffness parameter (West & Scafetta, 2003). The hopping model was specifically designed for modeling the firing intensities delivered by the central pattern generator (Ashkenazy et al., 2002; West & Scafetta, 2003, 2005). The neural centers that compose the central patterns generator were assumed to deliver impulses of particular frequencies, which are mutually correlated. These impulses are modeled by the nodes δ_i of a Markov chain that obeys a first-order autoregressive process:

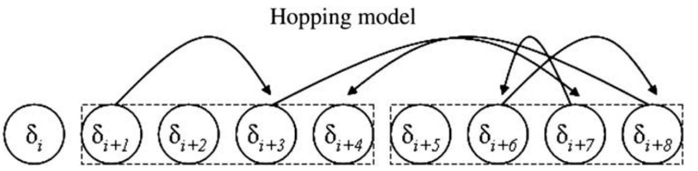

Fig. 2. Illustration of the hopping model. The dashed boxes represent correlated zones whose size r is related to auto-regressive parameter determining the Markov chain by $r = -1/\log\varphi$. The random walk activates successively the variable linear stiffness parameters δ_{i+1}, δ_{i+3}, δ_{i+7}, δ_{i+6}, δ_{i+8}, and δ_{i+4}, that are associated with the different neural centers.

$$\delta_i = \phi\, \delta_{i-1} + \varepsilon_i, \tag{1}$$

where $0 < \varphi < 1$ is a constant, and ε_i is a white noise process. These neural centers were moreover assumed to be randomly activated. This assumption is implemented as a random walk along the Markov chain, with jump sizes obeying a Gaussian distribution of width ρ (see Fig. 2). These random jumps, that gave the hopping model its name, thus generate a new series of values δ_n. These values correspond to the successively activated neural centers, and are injected into the van der Pol oscillator. The hopping model accounts for the long-range correlation evidenced in the stride interval series (West & Scafetta, 2003).

Although the processes that are engaged in the hopping model are quite simple – serial correlation basically arises from the combination of a random process and an autoregressive process – the model has been shown to generate genuine long-range correlation with systematic variations in intensity according to variations in parameters φ and ρ (Delignières et al., 2008). Regarding the theoretical interpretations of the hopping model, Ashkenazy et al. (2002) hypothesized that the range ρ of the random walk steps increases with neural maturation, accounting for developmental changes in the serial correlation of gait dynamics. Further studies showed that the parameterization of the external forcing function also explained the changes in serial correlation in normal, stressed, and externally paced gait conditions (West & Scafetta, 2003, 2005). Finally, $1/f^{\beta}$ noise has recently been found in the periods of unimanual self-paced oscillations. The dynamics of unimanual oscillations have commonly been modeled by a hybrid limit-cycle oscillator (Kay, Saltzman, Kelso, & Schöner, 1987). Delignières et al. (2008) proposed to inject the hopping model at the level of the oscillator's stiffness parameter, to account for the evidenced $1/f^{\beta}$ noise. According to the authors, the Markov chain could in that case be interpreted in terms of a chain of possible 'states' of the system, with neighboring states determined by similar factors and mutually correlated.

3.2.2. Shifting strategy model

Some models assume that processes show discrete transitions from one mode of operation (i.e., a specific mean or variance) to the next. These so-called regime switching models have been extensively studied in the field of econometrics and finance, and were shown to closely mimic long-range correlation (Diebold & Inoue, 2000; Gourieroux & Jasiak, 2001; Guégan, 2005; Smith, 2005). The behavior of many natural systems, including human performance, often presents such form of nonstationary (e.g., Gilden & Wilson, 1995a, 1995b). Local nonstationarities, i.e. changes in mean or variance that occur on relatively short time scales, are moreover typical for $1/f^{\beta}$ fluctuations. Examining serial correlations thus might reveal local nonstationarities as an integral part of persistent long-range correlation structures (for details on the relationships between nonstationarity and long-range correlation see Beran, Feng, Franke, Hess, & Ocker, 2003).

In order to account for $1/f^{\beta}$ noise in temporal estimation tasks, Wagenmakers et al. (2004) proposed a shifting strategy model. This model is an extension of the regime switching models and the classical activation-threshold models. First, it is assumed that, over the course of the temporal estimation task, participants repeatedly change strategies. During the time that they are in use, the different strategies are associated with particular threshold levels that determine the criterion amount of temporal information that has to be accumulated for a response. The threshold thus presents plateau-like variations in time. Second, the speed with which the accumulation process approaches the current threshold is assumed to vary between the successive estimations. The successive time intervals are given by the ratio between the threshold and the activation speed.

In the following section of this article, we propose the shifting strategy model as a unifying mechanistic account of the specific correlation structures evidenced in absolute and relative timing series, for different rhythmic movement tasks. For that reason we are going to detail the formal aspects of the model at that time. Nevertheless, we can already notice this model appears consistent with the non-stationarity observed in temporal estimation data (Madison, 2001), and that it was shown to generate long-range correlation (Wagenmakers et al., 2004).

3.2.3. Limitations of the mechanistic perspective

One possible pitfall of the mechanistic modeling perspective is that one may mistakenly believe that a good quantitative model fit equals qualitative or theoretical insight. It has often been pointed out that a good fit to the data is a necessary, but not a sufficient criterion for a model's usefulness (Roberts & Pashler, 2000). A consideration of a model's usefulness involves, for instance, also a consideration of the theoretical foundations of the model, a consideration of the extent to which the model points to new research directions, and a consideration of the generalizability of the model. When these aspects of a model start to play an important role, mechanistic models may potentially benefit by borrowing ideas that have been developed from within the nomothetic framework, although this is largely unexplored territory.

4. Mechanistic account of $1/f^{\beta}$ noise in absolute and relative timing

In order to illustrate the mechanistic perspective on $1/f^{\beta}$ noise, we now discuss the modeling of serial correlations in absolute and relative timing in self-paced tapping, synchronization tapping, and bimanual tapping.

Historically, models for absolute and relative timing have not been designed to account for serial correlations or $1/f^{\beta}$ noise – the explanation of the negative lag one autocorrelation within the Wing and Kristofferson absolute timing framework being the exception that confirms the rule. Nevertheless, empirical research has found $1/f^{\beta}$ noise to be present both in timing tasks such as unimanual tapping (e.g., Chen, Repp, & Patel, 2002; Gilden, 2001; Gilden et al., 1995; Lemoine, Torre, & Delignières, 2006; Yamada, 1995), and in relative timing that requires bimanual coordination (Torre et al., 2007).

Thus, although the current models for absolute and relative timing capture several characteristic features of human performance, they have thus far ignored the observed patterns of serial correlation, and, in particular, the presence of $1/f^{\beta}$ noise. Here we show that the current models can be extended to account for $1/f^{\beta}$ noise, and that this extension is consistent and straightforward. In particular, we let the shifting strategy model guide the behavior of an internal timekeeper. In self-paced tapping, this model extends the Wing and Kristofferson model (Wing & Kristofferson, 1973); in synchronization tapping, it extends Vorberg and Wing's linear phase correction model (Vorberg & Wing, 1996); and in bimanual tapping, it extends Ivry's multiple timer model (Ivry & Richardson, 2002). For each model, we compare the pattern of serial correlations that it generates to the pattern of serial correlations generated by human participants (see Appendix A for details).

4.1. Self-paced tapping

In self-paced tapping, participants are required to reproduce the rhythm of a metronome after it has stopped. Time interval series produced in self-paced tapping are often described by the Wing and Kristofferson model (Wing & Kristofferson, 1973). This model assumes a timekeeper that triggers successive motor responses (i.e., taps) by generating regularly spaced cognitive events. The execution of each tap is affected by a motor delay, so that the inter-response intervals IRI_n are given by

$$\mathrm{IRI}_n = C_n + M_{n+1} - M_n, \qquad (2)$$

where C_n is the time that it takes the timekeeper to generate a cognitive event and M_n is the motor delay. The model does not elaborate on the specific role of the timekeeper; the assumption is that C_n and M_n are uncorrelated white noise processes.

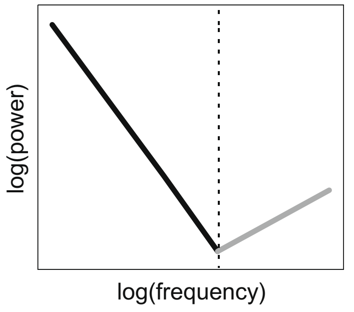

Fig. 3. Shape of the typical log–log power spectrum of inter-response interval series in self-paced tapping.

Gilden et al. (1995) were the first to report that IRI series show $1/f^{\beta}$ noise. Fig. 3 presents the typical shape of their power spectra. The characteristic shape of the spectrum is wedge-shaped with a negative slope, close to -1, in the low-frequency region, and a positive slope in the high-frequency region. The negative low frequency slope is though to reflect the contribution of a $1/f^{\beta}$ component, whereas the positive high frequency slope is thought to reflect a differenced white noise component (Delignières et al., 2004b; Delignières et al., 2008). According to the Wing and Kristofferson model, the differenced white noise in the IRI series originates from the difference between motor delays on successive trials (cf. Eq. (2)). This led Gilden et al. (1995) to suggest that the timekeeper is not a source of white noise, but rather a source of $1/f^{\beta}$ noise (Gilden, 2001; Gilden et al., 1995; Delignières et al., 2004b, 2008). The origin of the $1/f^{\beta}$ noise was not modeled.

4.1.1. Incorporating the shifting strategy model into the Wing and Kristofferson framework

In order to account for the long-range correlations in temporal estimation tasks, Wagenmakers et al. (2004) suggested a shifting strategy model. This model is an extension of the classical activation-threshold model (e.g., Ivry, 1996; Schöner, 2002) in which an activation process grows linearly in time until it reaches a particular threshold level. This threshold crossing determines a cognitive event that triggers the motor response and resets the activation process (see Durstewitz (2004), for neural plausibility). The shifting strategy model extends the activation-threshold mechanism in two ways.

First, as can be seen in Fig. 4a, the threshold level is not constant, but it is assumed to be affected by a sequence of "cognitive states" that causes plateau-like deviations of variable amplitudes and durations from its baseline level. For each of the successively adopted cognitive states, the amplitude T' of the threshold deviation is sampled from a uniform distribution of range R; this deviation is maintained for a duration d_n that is uniformly sampled from an interval $[d_{\min}; d_{\max}]$ of possible state durations. For each iteration, the current threshold is then given by:

$$T_n = T_0 + T'_n. \tag{3}$$

Second, the speed of the linear activation process is assumed to vary in an auto-regressive way around the baseline speed a_0:

$$a_n = a_0 + \phi(a_{n-1} - a_0) + \lambda\,\varepsilon_n, \tag{4}$$

where φ is the auto-regressive parameter, and ε_n a centered white noise with unit variance. As in the classical activation-threshold mechanism, the activation process is reset after it crosses threshold. Thus, the time it takes the timekeeper to generate a discrete cognitive event is determined by the ratio:

$$C_n = T_n/a_n. \tag{5}$$

The Wing and Kristofferson model can be easily combined with the shifting strategy model; all that is needed is to feed the timekeeper periods C_n (determined by Eq. (5)) into the Wing and Kristofferson model (i.e., Eq. (2)). Fig. 4b presents the complete model.

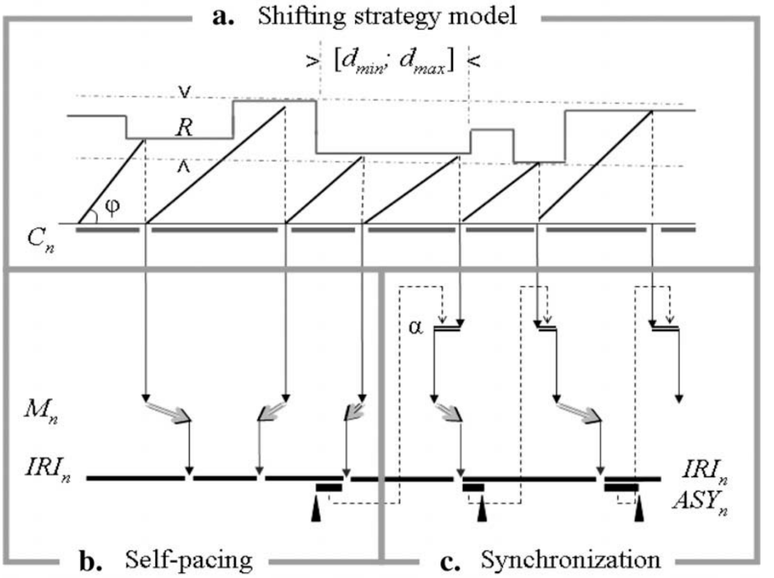

Fig. 4. Illustration of (a) the shifting strategy model, and its the incorporation into (b) the Wing and Kristofferson model for self-paced tapping, and (c) the Vorberg and Wing model for synchronization tapping.

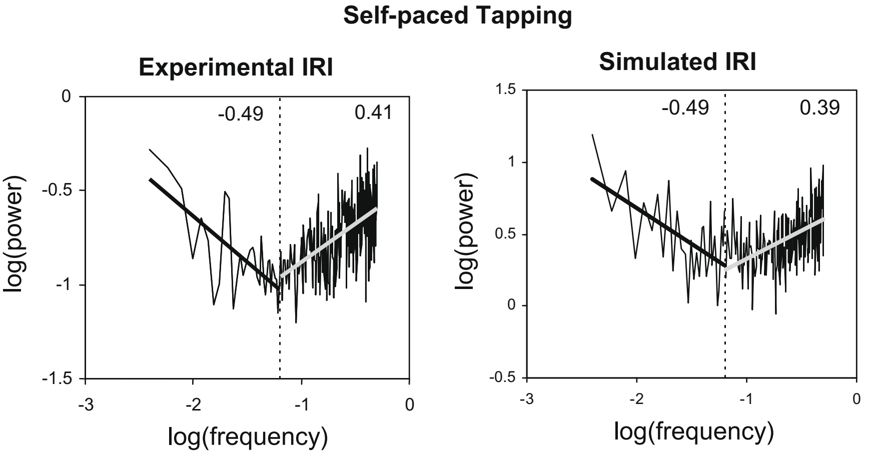

Fig. 5. Average log–log power spectra of experimental (left) and simulated (right) inter-tap interval series, in unimanual self-paced tapping. Parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 40$, $\varphi = 0.4$, and $\lambda = 0.08$ for the shifting strategy model. The standard deviation of the motor noise M was set to 20. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum (recall that the experiment featured 12 participants.)

4.1.2. Model performance for real data

The Wing and Kristofferson shifting strategy model provides a satisfactory account of self-paced tapping data. The mean of IRI series collected in a self-paced tapping experiment was 481 ms, with a mean standard deviation of 33 ms (data from Delignières et al., 2008; see Appendix A for details).

The mean of simulated IRI series was 501 ms, with an average standard deviation of 36 ms. As shown in Fig. 5, the model also reproduced the characteristic wedge-shaped log–log power spectra[2]; thus, the model captures both the $1/f^{\beta}$ component and the differenced white noise component (Delignières et al., 2008). Specifically, the mean slopes for experimental and simulated series were -0.49 and -0.49 in the low-frequency region of the spectrum, and 0.41 and 0.39 in the high-frequency region.

4.1.3. Discussion

The extended Wing and Kristofferson model provided a satisfactory account of the serial correlation pattern that is typical for self-paced tapping. The model is specific in the sense that the long-range correlations are thought to result from a shifting strategy process at the level of the timekeeper (see Delignières et al. (2008), for details).

It is likely that extensions of the Wing and Kristofferson model other than the shifting strategy model could also have produced $1/f^{\beta}$ noise. We prefer the shifting strategy extension because it is concrete, simple, and conceptually close to common physical representations of time (Schöner, 2002). Simulations of the shifting strategy model showed that variations in the values of the model parameters (R for the threshold deviations, and φ for the activation process) cause systematic variations in the intensity of long-range correlations. For extreme values, the long-range correlations are extinguished altogether (Delignières et al., 2008). The mechanisms of the model map on to factors that are likely to influence the functioning of the timekeeper (i.e., available attentional resources, number of cognitive strategies used, or the length of the target intervals), and this mapping allows the model to be further tested in a qualitative fashion.

The central assumption of the present model for self-paced tapping is that the source of $1/f^{\beta}$ noise is at the level of the timekeeper. If this assumption holds, $1/f^{\beta}$ noise would also have to be present in any other paradigm that involves the timekeeper. To illustrate this point we now turn to the paradigm of synchronization tapping.

4.2. Synchronization tapping

In the synchronization tapping paradigm, participants have to synchronize their taps with an external pacing signal that prescribes a regular tempo. In addition to an IRI series, the synchronization tapping paradigm also yields a series of asynchronies (ASY), which are defined as the time intervals between the participant's taps and the metronome's pacing signals.

In contrast to the IRI series in self-paced tapping, IRI series in synchronized tapping do not show $1/f^{\beta}$ noise – instead, these series show anti-persistent noise (i.e., negative correlations). The ASY series, however, do show $1/f^{\beta}$ noise (Chen et al., 1997, 2001, 2002; Ding et al., 2002; Torre & Delignières, 2008a). The IRI series and the ASY series actually contain similar information, since the IRI series correspond to the differentiation of asynchronies:

$$\mathrm{IRI}_n = \mathrm{ASY}_{n+1} - \mathrm{ASY}_n + \tau, \tag{6}$$

where τ is the constant period of the metronome. Thus, the anti-persistent noise in IRI series is the direct consequence of the presence of $1/f^{\beta}$ noise in the ASY series.[3]

One of the most used formal accounts of synchronization tapping is the linear phase correction model (Vorberg & Wing, 1996; see Repp (2005), for a review). This model contains a timekeeper that is active in both self-paced tapping and synchronization tapping. Just as the Wing and Kristofferson model, the linear phase correction model can be extended to account for $1/f^{\beta}$ noise by assuming that the timekeeper follows a shifting strategy process.

[2] Throughout this article, we quantify the serial correlations using the ${}^{\mathrm{low}}\mathrm{PSD}_{\mathrm{we}}$ method (Eke et al., 2000; see Appendix B for details).

[3] Differencing a $1/f^{\beta}$ time series creates a $1/f^{\beta-2}$ time series, so that a persistent time series with a slope of -1.1, say, becomes an anti-persistent time series with a slope of +0.9.

4.2.1. Incorporating the shifting strategy model into the linear phase correction framework

Vorberg and Wing's (1996) linear phase correction model assumes that the Wing and Kristofferson framework for self-paced tapping also applies to synchronization tapping. The Vorberg and Wing model accounts for synchronization tapping by a local correction of asynchronies, meaning that the timekeeper periods are supposed to be unaffected by the synchronization process (Semjen, Schulze, & Vorberg, 2000; Semjen, Vorberg, & Schulze, 1998; Vorberg & Schulze, 2002; Vorberg & Wing, 1996). In the model's initial and simplest formulation, each asynchrony between tap and metronome signal is corrected at the following tap by a first-order autoregressive or AR(1) process:

$$ASY_{n+1} = (1-\alpha)ASY_n + K_n - \tau, \tag{7}$$

where τ represents the period prescribed by the metronome. In this equation, K_n represents the inter-response intervals predicted by the original Wing and Kristofferson continuation model (IRI_n in Eq. (2)), that is, the intervals that would have been produced in the limit case where there is no effective synchronization process ($\alpha = 0$). The serial correlation in synchronization series thus result from the

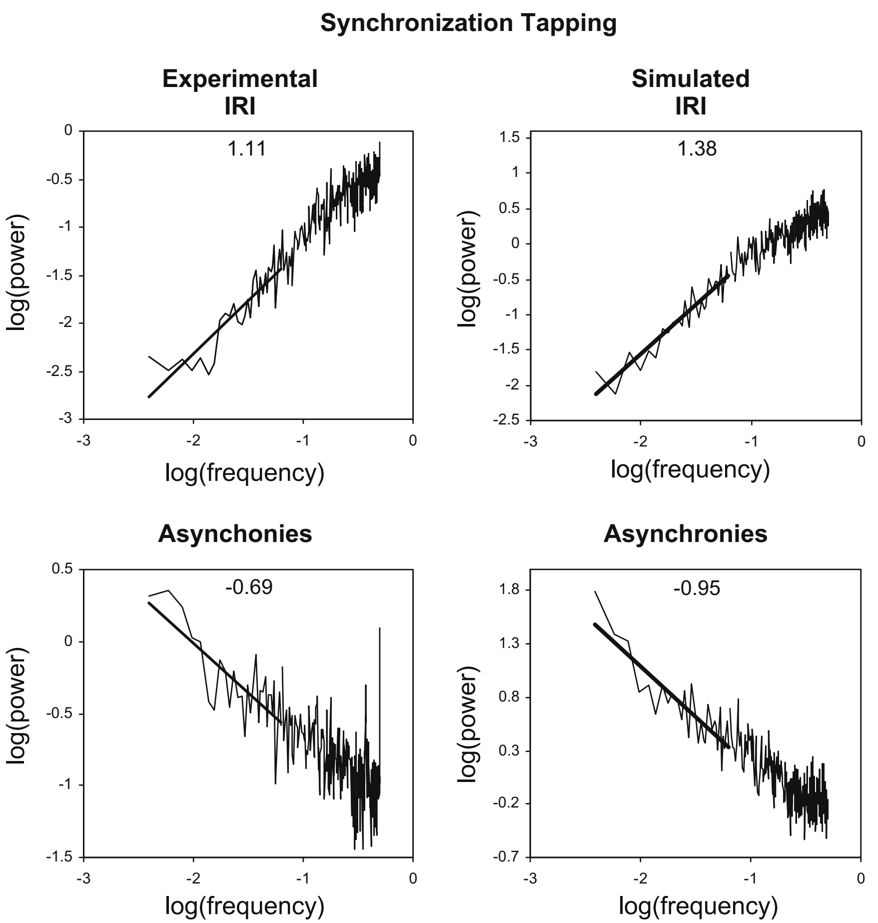

Fig. 6. Average log–log power spectra of experimental (left) and simulated (right) inter-tap interval series (top) and asynchronies (bottom), in unimanual synchronization tapping. Parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 40$, $\varphi = 0.4$, and $\lambda = 0.05$ for the shifting strategy model. The standard deviation of the motor noise M was 12 and the auto-regressive parameter α was set to 0.5. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum.

two-level architecture of the Wing and Kristofferson model, including the properties given to the timekeeper periods, plus the error correction processes.

Just as the original Wing and Kristofferson model, the Vorberg and Wing model conceptualizes the timekeeper as a source of uncorrelated white noise. This means that the model does not account for the correlation structures that have been observed in experimental ASY and IRI series (Torre & Delignières, 2008). Here we propose to extend the Vorberg and Wing model by incorporating the shifting strategy process at the timekeeper level. Fig. 4c provides a graphical illustration of the shifting strategy model for synchronization tapping.

Note that Vorberg and Schulze (2002) further proposed a more complex version of the linear phase correction model, including a second-order autoregressive term and a feedback delay on the perceived asynchronies. However, our present purpose was not to determine the best fitting and most complete model for synchronization tapping, but rather to show that, by letting the timekeeper process generate $1/f^{\beta}$ noise, we can produce a consistent account of the serial correlations observed in both self-paced and synchronization tapping.

4.2.2. Model performance for real data

The Vorberg and Wing shifting strategy model provides a satisfactory account of synchronized tapping data. The mean of IRI series collected in a synchronization tapping experiment was 499 ms, with a mean standard deviation of 32 ms, and the mean of experimental asynchronies was −62 ms, with a mean standard deviation of 35 ms. (see Appendix A for details on the experimental procedure). The mean of simulated IRI series was 500 ms, with an average standard deviation of 26 ms, and the mean of simulated asynchronies was 2 ms, with a mean standard deviation of 26 ms. Fig. 6 presents the average power spectra of experimental and simulated IRI and asynchrony series. For experimental series, the mean spectral slopes were 1.11 for IRI series, and −0.69 for ASY series. For the simulated series, the mean slopes were 1.38 for IRI series, and −.95 for ASY series.

4.2.3. Discussion

The extended linear phase correction model provided a satisfactory account of the serial correlation pattern that is typical for synchronization tapping. As was the case for self-paced tapping, the model assumes that the long-range correlations result from a shifting strategy process at the level of the timekeeper.

Our modeling efforts show that the hypothesis of a simple autoregressive correction in synchronization is consistent with the finding of $1/f^{\beta}$ noise in asynchronies, contrary to what was previously assumed (Chen et al., 1997; Pressing & Jolley-Rogers, 1997). The extended linear phase correction model should be easily testable within the synchronization tapping paradigm, in particular with respect to experimental factors – such as use of an auditory versus a visual metronome, or tapping on a contact surface versus air tapping – that are likely to influence the accuracy of the feedback on the produced asynchronies.

In the two absolute timing paradigms modeled so far, the timekeeper was assumed to function independently of the motor noise and the error correction process. That is, no constraint has been imposed on the shifting strategy component that generates the $1/f^{\beta}$ noise, and as a result the relationship between the correlations in the timekeeper series and the produced time interval series is relatively straightforward. We now consider a more complicated relative timing paradigm, in which two timers have been assumed to interact – the paradigm of bimanual tapping.

4.3. Bimanual tapping

In the bimanual tapping paradigm, participants have to produce a constant phase relationship between the movements of the two hands. Here we consider in-phase coordination, meaning that the right and the left taps have to coincide. In addition to the IRI series of the two hands (i.e., the component level), the bimanual coordination paradigm yields series of relative phase that describes the collective dynamics of the hand movements. The series of relative phase is defined as the time interval between the corresponding right and left taps, normalized by the completed IRI of the dominant hand.

The relative phase series has been the focus of most of the modeling work in bimanual coordination. Recently, this relative phase series was shown to display $1/f^{\beta}$ noise (Torre et al., 2007), a finding that speaks against the common prediction of a relative phase series that is white noise (e.g., Schöner, Haken, & Kelso, 1986). Within an experimental paradigm influenced by dynamical systems theory (Haken, Kelso, & Bunz, 1985), one could interpret the finding of $1/f^{\beta}$ noise as evidence for self-organized criticality. Considering a two-level analysis of the collective dynamics and the component dynamics, the structure of correlations in the relative timing pattern is determined by the congruence of the correlation structures of the two within-hand timing patterns. Thus, the correlation structure in the relative phase series can be assumed to be (at least partly) caused by processes determining the within-hand temporal patterns (Riley, Santana, & Turvey, 2001).

Current perspectives on the organization of bimanual coordination highlight the role of feed-forward timing (Ridderikhoff, Peper, & Beek, 2005), and support the hypothesis that similar timekeeping processes could be at work in bimanual coordination and unimanual timing tasks (Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Ivry, Richardson, & Helmuth, 2002; Semjen, 2002; Semjen & Ivry, 2001; Semjen & Summers, 2002). Ivry and collaborators (Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Ivry et al., 2002) extended the Wing and Kristofferson model for unimanual tapping to the paradigm of bimanual tapping. In Ivry et al.'s multiple timer model for bimanual tapping, a timer is responsible for the within-hand timing pattern of each hand, and a gating process implements a discrete temporal coupling that performs a sort of averaging of the two timers. A recent analysis of the serial correlation in the within-hand IRI series, however, showed that the absolute timing patterns in unimanual and bimanual tapping tasks are qualitatively similar (Torre & Delignières, 2008b). This finding suggests that by combining the shifting strategy model and the multiple timer model, one might be able to account for the serial correlations in both IRI and relative phase series that are observed in bimanual tapping.

4.3.1. Incorporating the shifting strategy model into the multiple timer framework

In the original multiple timer model, the timers associated with each hand were conceived as noisy activation-threshold mechanisms (Ivry & Richardson, 2002). The two timers produced white noise, and, in contrast to the unimanual case, they had no direct access to the effectors. A gating process ensures the temporal coordination by adding the thresholds (T_i and T_j) and the activation processes (a_i and a_j) of the two timers. When the integrated activation process reaches the normalized threshold, this marks the event in time that, in the case of in-phase coordination, triggers the taps of the two hands simultaneously. The time intervals prescribed by the gating process to the two hands are given by:

$$C = \frac{(T_i + T_j)}{(a_i + \sqrt{q}\varepsilon_i) + (a_j + \sqrt{q}\varepsilon_j)} \tag{8}$$

where ε is a white noise with variance q. According to the Wing and Kristofferson model, the execution of the taps was assumed to be affected by white noise. This original multiple timer model obviously does not account for long-range correlations, whether in the IRI series or in the relative phase series (Torre et al., 2007).

In order to account for the correlation properties of IRI and relative phase series in bimanual tapping, we propose to extend the multiple timer model in two ways. First, in order to account for the specific correlation structure in the effectors' IRI series, we replaced the noisy activation-threshold mechanisms that govern the behavior of the two timers by shifting strategy processes. However, since the multiple timer model assumes that the two timing processes merge into a single common signal for the two hands, the correlations in the relative phase series are only determined by the motor noise that affects the taps. That is, the model in its original configuration only generates white noise in the relative phase series, whatever the correlation properties of the time intervals generated by the two timers.

Thus, the goal of the second extension is to couple the two timing processes that account for correlations in the relative phase series, without altering the correlation properties of the IRI series. Therefore we proposed a continuous coupling of the thresholds, on the basis of their difference at each time:

$$T_{i,n} = T_{i,n} - \theta(T_{i,n} - T_{j,n})$$

$$T_{j,n} = T_{j,n} - \theta(T_{j,n} - T_{i,n}), \tag{9}$$

where θ is the coupling parameter ($0 < \theta < 0.5$). This coupling increased the congruence between the correlation structures of the two timekeeper series. Moreover, in order to prevent the divergence that would automatically occur between two noisy series, that is, to make stationary the delays between right and left taps, the onset of the activation process associated with the first tapping effector is assumed to await the completion of the tap of the second effector:

$$C_{i,n} = [T_{i,n} - \theta(T_{i,n} - T_{j,n})]/a_{i,n} + \delta_{j,i} \quad \text{for } \delta_{j,i} \geq 0, \tag{10}$$

$$C_{j,n} = [T_{j,n} - \theta(T_{j,n} - T_{i,n})]/a_{j,n} + \delta_{i,j} \quad \text{for } \delta_{i,j} \geq 0. \tag{11}$$

In this equation, a_n is the current activation speed of each timer as defined in the shifting strategy model (see Eq. (4)). δ is the delay between the two effective taps:

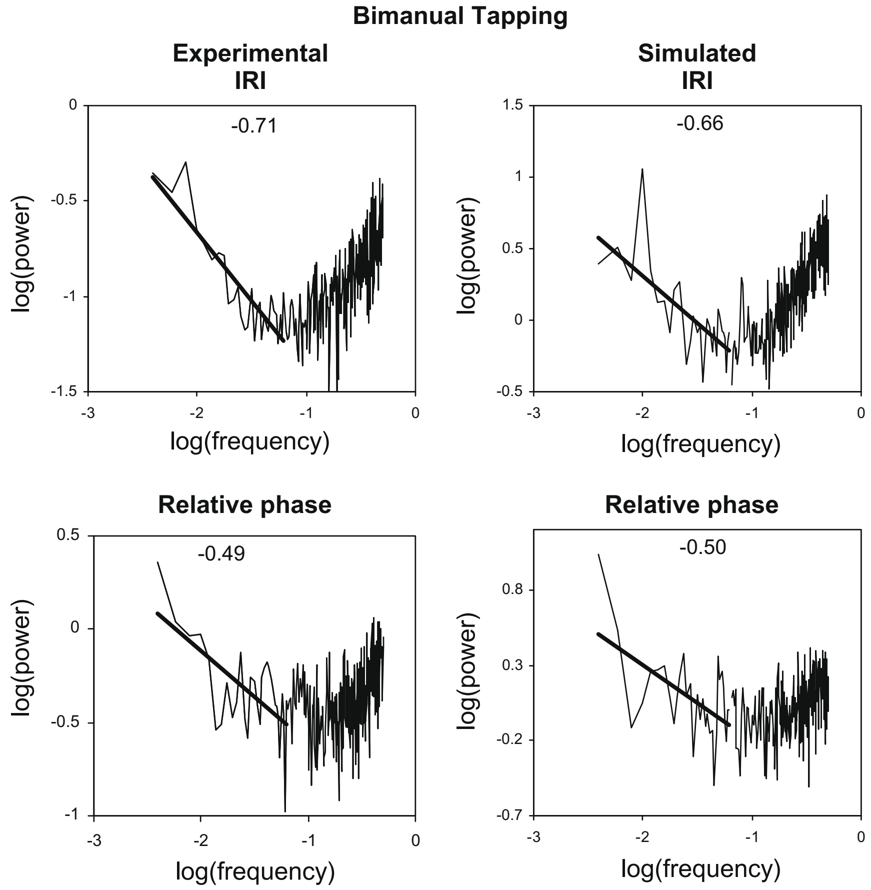

Fig. 7. Average log–log power spectra of experimental (left) and simulated (right) inter-tap interval series (top) of the right hand, and relative phase series (bottom) in bimanual in-phase tapping. For the two coupled timers, the parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 60$, $\varphi = 0.4$, $\lambda = 0.05$ for the shifting strategy model, and the standard deviation of the motor noise M was 12. The coupling parameter γ was set to 0.25. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum.

$$\delta_{j,i} = (C_{j,n} + M_{j,n}) - (C_{i,n} + M_{i,n}), \tag{12}$$

where M_n represents the random motor delay affecting each tap.

4.3.2. Model performance for real data

The model provides a satisfactory account of bimanual tapping data. The mean of IRI series collected in a bimanual tapping experiment was 470 ms, with a mean standard deviation of 30 ms, the two hands taken together. The mean of experimental relative phase series was $-12°$, with a mean standard deviation of 53° (see Appendix A for details on the experimental procedure). The mean of simulated IRI series was 511 ms, with an average standard deviation of 30 ms, and the mean of simulated relative phase series was 0°, with a mean standard deviation of 22°. Fig. 7 presents the average power spectra of experimental and simulated IRI and relative phase series. For experimental series, the mean spectral slopes were -0.71 for IRI series, and -0.49 for the relative phase series. For the simulated series, the mean slopes were -0.66 for IRI series, and -0.50 for the relative phase series.

4.3.3. Discussion

The extended multiple timer model provided a satisfactory account of the serial correlation pattern that is typical for bimanual tapping, both for the within-hand IRI series and for the relative phase series. The nature of the extension was twofold. First, just as in the case of unimanual tapping, we assumed that the two timers are the sources of $1/f^{\beta}$ noise, and we modeled their behavior through the shifting strategy model. Second, we determined the conditions of coupling the two shifting strategy models under which the correlation structures of both the IRI and the relative phase series matched the experimental correlations.

This development led us to formulate the coordination in bimanual tapping as a parallel organization with continuous interaction between the two coupled timers (as for example the HKB coupled oscillator model; Haken et al., 1985), instead of the sequential organization assumed in the original multiple timer model, where a gating process locked the two timing process. Our present model first assumed a coupling of the thresholds of the two timers. This seems consistent with the above consideration that the evolution of the threshold in the shifting strategy model was related to factors as cognitive strategies or the target intervals to produce, since these factors can be assumed common to the two timers. Second, it was assumed that the onset of the activation process associated to the earliest of the two tapping effectors awaits the effective tap of the second effector. This effectively makes stationary the delays between the two taps. Moreover, this functioning could be easily extended to anti-phase tapping, by assuming that the activation processes of the two timers grow two times faster. In that case, the between-hand intervals would be regulated instead of the within-hand intervals, as suggested in earlier studies (e.g., Semjen, 2002; Semjen & Ivry, 2001).

Further experimental testing of this model appears more constraining than for the self-paced or synchronization models, since each factor would simultaneously influence the correlation structures of the absolute and the relative time interval series. Consider the example of directing attention on one of the two effectors. When one assumes that the autoregressive variations of the activation process in the shifting strategy model maps on to attentional fluctuations, one can expect that directed attention would increase the discrepancy between the correlation structures produced by the two timers. We computed the spectral coherence between the right and left IRI series collected in the present bimanual tapping experiment. The analysis showed very high coherence coefficients, with a mean r^2 of about .94 (SD = .08). In the condition of directed attention, this coherence coefficient should decrease. Moreover, since the correlation structure in the relative phase series is directly related to the coherence between the correlation structures of the two IRI series, the persistent correlations in the relative phase should increase in the same time.

5. Concluding comments

The aim of this paper was to show how a mechanistic perspective on $1/f^{\beta}$ noise can advance theories of human movement production. We argued that domain-specific models are useful to establish a clear link between $1/f^{\beta}$ noise on the one hand and the substantive psychological and/or biological phenomenon on the other. We supported our claim by applying the shifting strategy model (e.g., Wagenmakers et al., 2004) to three standard tasks in the field of human rhythmic movement production: self-paced tapping, synchronization tapping, and bimanual tapping. In all cases, we extended current models by assuming that the timekeeper undergoes a shifting strategy process.

It should be clearly acknowledged that nomothetic accounts of $1/f^{\beta}$ noise – those that seek general explanations and refer to concepts such as complex systems, emergent dynamics, metastability, and self-organized criticality – that such accounts certainly point to universal principles that produce $1/f^{\beta}$ noise in number of systems and phenomena which superficially have little in common. However, the general focus of the nomothetic accounts can sometimes make it difficult to respect and account for the idiosyncrasies of $1/f^{\beta}$ noise processes in specific applications.

A challenge for the nomothetic account is to handle the fact that $1/f^{\beta}$ noise is not observed always and everywhere (e.g., series of asynchronies from the present experiment on synchronization tapping contained $1/f^{\beta}$ noise, whereas periods did not). The intensity of $1/f^{\beta}$ noise is often sensitive to particular experimental manipulations, and may even be absent altogether (e.g., Chen et al., 2001; Hausdorff et al., 1996; Jordan et al., 2006; Jordan et al., 2007; Madison, 2001, Madison, 2004). Clearly, it is unlikely that, as a result of these experimental manipulations, the human brain has ceased to be complex, multileveled, or metastable. This challenge for the nomothetic account may be more apparent than real; perhaps concrete modeling of the phenomena under consideration will confirm that the nomothetic concepts such as SOC are sensitive to the same manipulations that influence the intensity of $1/f^{\beta}$ noise in an experiment.

It may be possible to achieve a resolution between the nomothetic and mechanistic perspectives on $1/f^{\beta}$ noise by arguing that these perspectives operate on different levels of explanation. As we have hinted at throughout this article, the development of concrete, domain-specific models is not fundamentally at odds with the claim that $1/f^{\beta}$ noise originates from some universal principle. For instance, both in the hopping model and in the shifting strategy model, $1/f^{\beta}$ noise comes about through the combination of a simple autoregressive process and a regime switching process – that is, in both cases $1/f^{\beta}$ noise originates through the aggregation of processes that operate on different time scales.

In sum, the two perspectives on $1/f^{\beta}$ noise would gain in being considered complementary, the strength of the one defining the limitation of the other one. Nevertheless, the impact of $1/f^{\beta}$ noise on human movement science depends to a large extent on which perspective one adopts to account for the phenomenon. Domain-specific mechanistic models can not pretend to uncover the universal principles that account for the ubiquity of $1/f^{\beta}$ noise, as nomothetic accounts can do. In contrast, mechanistic accounts offer the advantages of specific, experimentally testable and thus falsifiable models of human behavior. Regardless of which perspective on $1/f^{\beta}$ noise one prefers, it is clear that current models of human behavior have not been designed to account for serial long-range correlations. But the $1/f^{\beta}$ noise phenomenon is there, and its presence constitutes a constraint that should be taken into account. Our article shows how one can model the presence and intensity of $1/f^{\beta}$ noise in human movement science in a way that is generalizable, testable, and quantitatively precise.

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Appendix A. Experimental procedures

Twelve participants (mean age 29 ± 7 yrs) performed series of 600 taps (trial durations about 5 min) with their dominant hand index finger(s) on a flat pressure sensor disposed on a table. Three experimental conditions were randomly assigned.

(1) In the self-paced tapping condition, the target time interval was 500 ms. This interval was presented in a continuation paradigm: a 30-s video displayed the task to perform at the required

tempo, and the participants had to reproduce the tempo as accurately and regularly as possible over the task duration, immediately after watching the video. We analyzed the produced inter-response interval (IRI) series.

(2) In the synchronization tapping condition, a PC-driven metronome delivered acoustic signals with a constant period of 500 ms, and the participants were instructed to synchronize their taps with the metronome. We analyzed the IRI series and the series of asynchronies, defined as the differences between the times of the effective taps and the signals of the metronome.

(3) In the bimanual tapping condition, participants had to perform in-phase coordination, i.e., simultaneous taps, without an external pacing signal. Participants were instructed to be as accurate and regular as possible in the coordination of the taps and in the tempo of movements. We analyzed the absolute timing patterns of the hands with the IRI series, and the relative timing pattern with the relative phase series.

Appendix B. Data analyses

We analyzed experimental and simulated time series of 512 points. The power spectra were computed using the ${}^{low}\mathrm{PSD}_{we}$, initially developed by Eke et al. (2000). The method includes three preprocessing operations before performing the Fast Fourier Transform. First the mean of the series is subtracted from each value. Second, a parabolic window is applied to taper the series. Third, a linear detrending is performed on the entire series. Finally, in order to obtain a more accurate estimate of the spectral slope, Eke et al. perform a linear regression only on the low-frequency region of the log–log power spectrum. The low frequencies were defined as $f < 1/8$ of the maximal frequency that composes the signal. We also used this same boundary frequency to divide the spectrum in a low-frequency and a high-frequency region in case separate slope estimates were required.

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